METRIC SPACES AND TOPOLOGY

Spring 2023

LECTURER
Anush Tserunyan
(McGill University)
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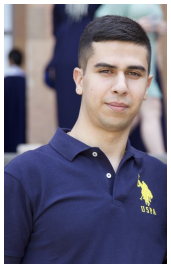
LECTURE
Tue & Fri 17:00–18:20



TEACHING ASSISTANT
Aram Bughdaryan
(Yerevan State University)
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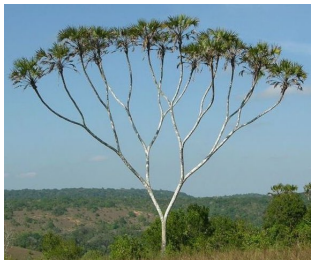
PROBLEM SESSION
Tue & Fri 18:30–19:30

LOCATION: Institute of Mathematics, Hall 1.

LANGUAGE: The lectures and problems will be written in English, but **spoken in Armenian**.

TO REGISTER: Email anush.tserunyan@mcgill.ca your full name and university affiliation, for example: 2nd year undergraduate at Informatics and Applied Math, YSU.

COURSE DESCRIPTION. Metric spaces are sets with a given *metric*, i.e. distance function. For example, $\mathbb{R}$ with the distance between points x, y being $|x-y|$, or the set $C([0,1])$ of continuous functions on $[0,1]$ with the distance between functions f, g being $\sup_{x\in[0,1]} |f(x) - g(x)|$. Metric measures how different two objects are and it can be used to understand how objects are located with respect to others. Most importantly, metric yields a notion of convergence, which allows for proving the existence of a desirable object (e.g. a solution to a PDE) by only building approximations to it.



Cantor set

Besides analysis on metric spaces, we will also study their generalization, namely, *topological spaces*, where the notion of closeness need not satisfy the "triangle inequality." Social distance is such an example: your friends know you, you know your coworkers, but your friends may not know your coworkers. The course topics will include:

- Open sets in metric spaces and convergence
- Complete metric spaces and completion
- Separability and compactness
- Continuity and uniform convergence
- Baire category theorem and applications
- Topological spaces
- Bases and generation, product topology
- Connectedness and separation axioms
- Continuous functions and extensions

PREREQUISITES: It is **not required** to have taken our Honours Analysis course last semester, but familiarity with real analysis (on $\mathbb{R}$) is necessary.

METHOD OF EVALUATION: There will be weekly homework assignments, with solutions presented at the board by the students in problem sessions. We will also have a midterm and a final exam. The grade will be determined by **35% homework + 25% midterm + 40% final**.

COURSE MATERIAL:

- I. KAPLANSKY, *Set Theory and Metric Spaces* (2nd ed.), AMS Chelsea Publishing, 1957.
- T. GAMELIN, R. GREENE, *Introduction to topology* (2nd ed.), Mineola, N.Y., Dover, 1999.
- G. FOLLAND, *Real Analysis: Modern Techniques and Their Applications* (2nd ed.), Pure and Applied Mathematics, N.Y., John Wiley & Sons, 1999.